Teachers notes

Integration: finding the tangent and normal commentary

The resource: There are 10 sets of matching cards each with 11 cards in. Students should first identify the question. They will then be left with 5 cards that show the working out at each stage and 5 cards that give a commentary of what needs to be done. There are also worksheets for students to write a commentary on a similar problem where they need to find the normal.

Suggested use: This is a good way of getting students started on questions about finding the tangent and the normal as it breaks the steps down and makes them think about what happens at each stage. It also provides good revision notes as otherwise examples can be confusing. Once they have been able to correctly match the tangent cards they should move on to writing a commentary for the working out give to find a normal.They may be reluctant or unsure about what to write so it may help to leave the tangent matching cards out for inspiration.

Notes for printing: The first page gives the matching cards for finding the tangent to a curve, they are not in order and so can be given out to students to cut up if you want them to be able to keep a copy. The 2nd and 3rd pages are the same so that they can be printed 2 to a page and given to students.

Substitute the x into the gradient formula to find the gradient of the tangent.	$$y = \frac{(-2) - 2}{(-2)^2}$$
Expand and simplify the function so it is easier to differentiate.	$$y = \frac{-4}{4}$$ $$y = -1$$
Use the formula $y - y_1 = m(x - x_1)$ to find the equation of the line.	$$y = \frac{x - 2}{x^2}$$ $$y = (x - 2)x^{-2}$$
Differentiate using the rule take one off the power and multiply by the original power.	$$y = x^{-1} - x^{-2}$$
Substitute the x value into the original equation to get the y value.	$$m = -(-2)^{-2} + 4(-2)^{-3}$$ $$m = -\frac{1}{4} + 4\left(\frac{1}{-8}\right)$$ $$m = -\frac{1}{4} - \frac{4}{8}$$
Find the tangent to the curve $y = \frac{x-2}{x^2}$ at x = -2	$$m = -\frac{1}{4} - \frac{2}{4}$$ $$m = -\frac{3}{4}$$
$$\frac{dy}{dx} = -x^{-2} + 4x^{-3}$$	$$y - (-1) = -\frac{3}{4}(x - (-2))$$ $$4(y + 1) = -3(x + 2)$$ $$4y + 4 = -3x - 6$$ $$3x + 4y + 4 + 6 = 0$$ $$3x + 4y + 10 = 0$$

Find the normal to the curve $y = \frac{16}{x^2} - \sqrt{x}$ at the point (4,-1)

Working out	Commentary
$$y = 16x^{-2} - x^{\frac{1}{2}}$$	
$$\frac{dy}{dx} = -32x^{-3} - \frac{1}{2}x^{-\frac{1}{2}}$$	
$$m = -32(4)^{-3} - \frac{1}{2}(4)^{-\frac{1}{2}}$$ $$m = -\frac{32}{64} - \frac{1}{4}$$ $$m = -\frac{3}{4}$$	
$$m = \frac{4}{3}$$	
$$y - (-1) = \frac{4}{3}(x - 4)$$ $$3y + 3 = 4x + 16$$ $$3y = 4x - 19$$ $$4x - 3y - 19 = 0$$	

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