Power Hour

A botany student counted the number of daisies in 42 areas of a field. The results are summarised in the following stem & leaf diagram.
(a) Write down the modal value of these data. **(1)**
(b) Find the median and the quartiles of these data. **(4)**
(c) On graph paper and showing your scale clearly, draw a box plot to represent these data. **(4)**
(d) Comment on the skewness of this distribution. **(1)**

Number of daisies									1 | 1 means 11
1	1	2	2	3	4	4	4		(7)
1	5	5	6	7	8	9	9		(7)
2	0	0	1	3	3	3	3	4	(8)
2	5	5	6	7	9	9	9		(7)
3	0	0	1	2	4	4			(6)
3	6	6	7	8	8				(5)
4	1	3							(2)

a) 23

b) $\frac{n}{4} = 10.5 = 11^{th} = 17$

$\frac{n}{2} = 21^{st} = \frac{23+24}{2} = 23.5$

$\frac{3n}{4} = 31.5 = 31$

d) $Q_2 - Q_1 = 6.5$

$Q_3 - Q_2 = 7.5$

slight positive skew

A company wants to pay its employees according to their performance at work. The performance score x and the annual salary, y in £100s, for a random sample of 10 of its employees for last year were recorded. The results are shown in the table below.
(a) Calculate exact values of S_{xy} and S_{xx}. **(4)**

x	15	40	27	39	37	15	20	30	19	24
y	216	384	234	399	226	132	175	316	187	196

[You may assume $\Sigma y^2 = 69\,798$, $\Sigma x^2 = 7\,266$]

a) $S_{xy} = 69798 - \frac{256 \times 2465}{10} = 6694$

$S_{xx} = 7266 - \frac{256^2}{10} = 712.4$

(b) Calculate the equation of the regression line of *y* on *x*, in the form $y = a + bx$. Give the values of *a* and *b* to 3 significant figures. **(5)**

b) $b = \frac{6694}{712.4} = 9.3964...$

$a = \frac{2465}{10} - \frac{6694}{712.4} \times \frac{256}{10} = 5.95199...$

$y = 5.95 + 9.40x$

The mark, *x* scored by each student who set a statistics exam is coded using $y = 1.4x - 20$. The coded marks have mean 60.8 and standard deviation 6.60. Find the mean and standard deviation of *x*.

mean $= 1.4(60.8) - 20 = 65.12$

$\sigma = 1.4(6.60) = 9.24$

Two events *A* and *B* are independent, such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Find
(a) $P(A \cap B)$, **(1)**
(b) $P(A|B)$, **(1)**
(c) $P(A \cup B)$. **(1)**

a) $P(A \cap B) = P(A)P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

b) $P(A|B) = \frac{1}{3}$

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$

The events *A* and *B* are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Represent these probabilities in a Venn diagram.

A: $\frac{1}{4}$, $A \cap B$: $\frac{1}{4}$, B: $\frac{1}{12}$, outside: $\frac{5}{12}$

One of the objectives of a computer game is to collect keys. There are three stages to the game. The probability of collecting a key at the first stage is $\frac{2}{3}$, at the second stage is $\frac{1}{2}$, and at the third stage is $\frac{1}{4}$.
(a) Draw a tree diagram to represent the 3 stages of the game. **(4)**
(c) Find the probability of collecting exactly one key in a game. **(5)**

$P(K, K', K') + P(K', K, K') + P(K', K', K)$

$= \frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{8} + \frac{1}{24} = \frac{5}{12}$

Cooking sauces are sold in jars containing a stated weight of 500 g of sauce. The jars are filled by a machine. The actual weight of sauce in each jar is normally distributed with mean 505 g & standard deviation 10 g.
(i) Find the probability of a jar containing less than the stated weight.
(ii) In a box of 30 jars, find the expected number of jars containing less than the stated weight. **(5)**

$\sigma = 10$

i) $P(X < 500)$, $z = \frac{500 - 505}{10} = -0.5$

$1 - 0.6915 = 0.3085$

ii) $30 \times 0.3085 = 9.255$

A discrete random variable *X* has the probability function shown in the table.
Find (a) $P(1 < X \leq 3)$, **(2)** (b) $F(2.6)$, **(1)** (c) $E(X)$, **(2)** (d) $E(2X - 3)$ **(2)** (e) $Var(X)$ **(3)**

x	0	1	2	3
P(X=x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{12}$

a) $P(2) + P(3) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

b) $F(2.6) = P(X \leq 2) = \frac{1}{3} + \frac{1}{2} + \frac{1}{12} = \frac{11}{12}$

c) $E(X) = (0 \times \frac{1}{3}) + (1 \times \frac{1}{2}) + (2 \times \frac{1}{12}) + (3 \times \frac{1}{12}) = \frac{11}{12}$

d) $2E(X) - 3 = 2\left(\frac{11}{12}\right) - 3 = -\frac{7}{6}$

e) $Var(X) = 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{12} + 3^2 \times \frac{1}{12} - \left(\frac{11}{12}\right)^2 = \frac{107}{144}$